MODELLING AND ANALYSING THE TENSILE AND SHEARING BEHAVIOUR OF FABRIC SAMPLES

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Abstract

Textiles and woven fabrics have widely been used in different constructions such as covering sheets or fibre reinforced polymer composites. The poster is about a fibre-bundle-cells based structural-mechanical model for describing the tensile- and shearing behaviour of woven fabrics. The modelling method is based on some idealized statistical fibre-bundle-cells as model elements, developed earlier, and the parallel and serial connections of them. For phenomenological applications modelling software FibreSpace has been used. The simplified FBC model of the fabric samples made it possible to study and analyse the effects of the cutting angle and the place of the critical cross section. In the case of instantaneous failure mode the simplified FBC model without any structural correction strongly underrated the mean breaking force for the cutting angles between 15° and 75° while this model with correcting the yarn orientation and the shearing effects related adhesion provided good fitting between 30° and 60° . These results give an important contribution to realizing the correct FBC models of the fabric samples in the FibreSpace program, in which the deformation, crosswise contraction, damage, and failure as time dependent processes are included.

Introduction

Woven fabrics have widely been used in human and high-tech applications or as reinforcement in polymer composites. The engineering applications need designing and dimensioning based on knowledge on relationships between structure and mechanical properties [1-3]. In general, fabrics are built up of two yarn or roving systems perpendicular to each other. Samples cut out in any main direction of fabrics consist of aligned warp and weft yarns creating yarn bundles and a section of a yarn between two crossing points creates a fibre bundle. The fibre and yarn bundles as kind of intermediate elements represent the statistical properties of the fibrous structure and its strength such as deformation behaviour, damage process, failure and breakage, and size effects [1, 3-6]. On the basis of that the so called statistical fibre-bundle-cells (FBC) method and software FibreSpace have been developed by the authors [7-15].

The FBC modelling method is based on some idealized fibre bundles called fibre-bundlecells (FBCs) which can be used as building elements of a model network like those in the viscoelastic mechanical models such as spring and dashpot [8-12]. FBCs are defined as fibre classes containing the same geometrical (shape, disposition) and mechanical properties (strain state, gripping by the environment) (Figure 1). In the simplest case the fibres are ideally elastic (E) with linear relationship between the strain and tensile force (F=Kε, K is the tensile stiffness) however, they break at a random breaking strain $(\epsilon_{\rm s})$. The fibres of E-bundle (classic fibre bundle [3-6]) are straight and parallel to the tensile load direction and they are ideally gripped that is they do not slip out of the grips. In the other 3 bundles some of the ideal properties are hurt like the fibres may be crimped or pretensioned (ϵ_0) in the EH-bundle or oblique in the ET-bundle, as well as they can slip out of the grips or possible connections in the ES-bundle. In the latter case the slipping fibre transmits tensile force referred to e.g. the adhesion resistant ($F_b \leftrightarrow \varepsilon_b$) until reaching a random slippage length (ε_{bL}). Both the geometrical and the strength parameters of fibres are assumed to be independent stochastic variables [7-9]. All these properties can be combined in the EHT, EST or EHST bundles.

Figure 1 Schematic of the linear FBC-s and the relationships between the strains of bundle (u) and fibres $(ε)$ [14]

The weighted parallel connection of them can provide an adequate FBC model for describing the mechanical behaviour of a fibrous sample during tensile test. On the basis of this model the deformation and damage processes of fibres and yarns within a fibrous sample can be studied and analysed. FibreSpace can give a hand in the creation of FBC network models.

In the case of the fabric the yarns play the role of the fibre elements, however, they have got nonlinear tensile characteristic therefore modelling a bundle of them needs nonlinear FBCs [14]. Samples cut out in any main direction of fabrics consist of aligned warp and weft yarns creating yarn bundles. Samples cut out any other direction have more complicated bundle structure [14].

In this paper non-linear ET-bundles and special simplified EST-bundles using instantaneous failure mode were applied to modelling the mean tensile strength of fabric specimens cut out of different directions and analysing certain shearing effects.

Fibre-Bundle-Model of Fabric

Cutting out a sample e.g. in warp direction from the fabric is built of warp yarns aligned lengthwise and weft yarns aligned crosswise (Figure 2). Loading this sample in lengthwise direction the load is taken up by the warp yarns gripped at both ends and the weft yarns with free ends play just a modifying role by interlacing and crimping the warp yarns. This specimen is considered as an equivalent E-bundle created by fabric yarn elements which describe the mean tensile process of the fabric specimen including the effects of the yarnyarn adhesion and the possible slippage of the weft yarns.

Figure 2 Modelling fabric sample of main direction as an equivalent yarn element bundle

The expected value of the tensile force process of the nonlinear E-bundle can be calculated by the following formula [14]:

$$
\overline{F}(u; p) = E[F(u; p)] = f(u; p_1)[1 - Q_{\lambda_S}(u; p_2)]
$$
\n(1)

where u is the bundle elongation, $Q_{\lambda S}$ is the distribution functions of the breaking elongation $(\lambda_{\rm S})$ of the model yarns, while $f(u)$, $f(0)=0$, is the normalized tensile characteristic of the model yarns, which is nonlinear in general, and parameter vector $p=(p_1, p_2)$ denote the parameters of $f(u)$ and the distribution function of $\lambda_{\rm S}$. The components of vector p_2 are the expected value, and the standard deviation. In Equation (1) the tensile characteristic, $f(u)$, describes the failureless work of the fibrous structure while $1-Q_{\lambda S}(u)$ is a kind of reliability function and represents the statistical properties of the damage process. In this case the following rather flexible function was applied to the description of the tensile characterisctic of the model yarns:

$$
f(u; p_1) = c \left(u - \lambda_o + (\lambda_o - \lambda_1) e^{-\frac{u - \lambda_1}{\lambda_o - \lambda_1}} \right) \Theta(u - \lambda_1)
$$
 (2)

where $p_1=(c,\lambda_0,\lambda_1)$, λ_0 and λ_1 are respectively the possible preextension and shifting parameters, $c=K$ is the (asymptotic) tensile stiffnesses of the model-yarn, while θ is the unitstep function. In addition the breaking elongation of the model yarn elements (λ_s) was assumed to be of normal distribution of parameter p_2 .

A yarn element in a rectangle-like fabric samples cut out of different directions can be gripped at both ends (2-gripped), or just at one of its ends (1-gripped: 1' or 1" for gripped at the left or right end, respectively), or at none (0-gripped) (Figure 3.a). The yarns intersecting a cross section x in a fabric sample with cutting angle α (Figure 3.b) consist of right and left parts the length of which are the so called right (*l*⁺) and left (*l*) beard-lengths [9]. Assuming that the yarn-yarn interaction based resistant force of a yarn against pulling out is proportional to its gripped length the so called critical adhesion length (l_s) can be defined as a gripped length at which this resistant force equals the breaking force of the yarn (F_S) :

n

$$
F_S = f_b l_S \tag{3}
$$

where f_b is the specific resistant force. Consequently, a yarn intersecting the weakest cross section (x), where failure of the sample takes place, breaks if its so called active beard length (*la*) is equal to or larger than the critical adhesion length otherwise slips out of the grip realized by its vicinity in the sample [9]. The active beard length depends on the grip mode of the yarns (Figure 3.b):

Figure 3 Possible gripping modes of yarns in a textile specimen of length L_0 and width B_0 (a) right and left beard lengths of yarns gripped in different ways (b)

Because of the slippage for the mathematical description of mechanical behaviour of the 1 or 0-gripped yarn the special version of the ES- or EST-bundles are to be used since the resistant force of a yarn depends on its vicinity-gripped length while they are considered as independent of each other in the original ES-bundle [9]. This special ES-bundle is denoted by ES2 or its oblique version by ES2T.

Disregarding the main directions Figure 4 shows the cutting angle dependent possible bundle structures of a fabric sample where α_1 and α_2 are the diagonal direction of the sample the warp and weft yarns are marked by red and blue, respectively.

The expected value of the tensile force measured at tensile testing a fabric sample cut out in direction α is the sum of the resistances of the warp and weft yarns in the load direction (i∈{warp, weft}, j∈{2, 1', 1", 0}):

$$
E(F(u, \alpha, x)) = E(F_{warp}(u, \alpha, x)) + E(F_{weff}(u, \alpha, x)) = \sum_{i} n_i(\alpha) \sum_{j} P_{ij}(\alpha, x) F1_{ij}(u, \alpha, x)
$$
(5)

where the number of yarns intersecting any cross section can be calculated from the yarn densities (A_{i}) and the sample width (B_{o}) :

$$
n_i(\alpha) = \begin{cases} \Lambda_i B_o \cos \alpha, & i = warp \\ \Lambda_i B_o \sin \alpha, & i = weft \end{cases}
$$
 (6)

while $F1_{ij}$ is the force projection referred to one yarn and P_{ij} is the yarn number fraction in cross section 0≤x≤L_o:

$$
P_{ij}(\alpha, x) = \frac{n_{ij}(\alpha, x)}{n_i(\alpha)}
$$
\n(7)

It should be noted that the tensile force is the same in every cross section however the failure process depends on the place (x) where it takes place. The tensile strength of a fabric sample is the minimum of the breaking forces given by the maximum tensile forces along the sample length:

$$
\overline{F}_{Sf}(\alpha) = \min_{x} \left(\max_{u} E(F(u, \alpha, x)) \right)
$$
 (8)

In Figure 5 the bundle structure of a sample (Figure 5.a) and the variation of the yarn number fractions calculated according to Equation (7) along the length can be seen.

Figure 5 Bundle types of the warp (red) and weft (blue) yarns at cutting angle $\alpha_1 < \alpha < \alpha_2$ (a), number fraction of yarns of different gripping in the cross sections of a fabric specimen cut in direction $\alpha_1<\alpha=35^\circ<\alpha_2$ (b)

The yarn force projection, F1_{ij}, is calculated for 2-gripped yarns with ET-bundle while for 1- or 0-gripped yarns with ES2T-bundles. The latter are different according to the distribution of the active beard length given by Equation (4). These distributions were derived and used for the further calculations. The FBCs provide the total expected tensile force process including

the time dependent damage process up to the failure of the last yarn. According to Equation (5) this means the calculation of the resultant force of 8 parallel connected FBC. These calculations can strongly be simplified by supposing that all the breakages and slippages of yarns in the failure cross section take place at the same instant realizing a catastrophic ultimate failure. In this case the tensile strength of the sample can be calculated by taking into account and summing up the breaking and slipping resistant forces of yarns intersecting the given cross section. The calculation of the expected value of $F1_{ii}$ can be performed by the following formula (i∈{warp, weft}, j∈{2, 1', 1", 0}):

$$
E(F1_{ij}(\alpha, x)) = F_{SL,i} \left[\frac{1}{l_{Si}} \int_{0}^{l_{Si}} y dQ_{l_{ij,a}}(y, x) + \left(1 - Q_{l_{ij,a}}(l_{Si}, x)\right) \right]
$$
(9)

where $Q_{li,a}(y,x)$ is the distribution function of active beard length, $l_{li,a}$, and $F_{SL,i}$ is the lengthwise projection of the mean breaking force of the yarns:

$$
F_{SL,i} = \begin{cases} F_{Si} \cos \alpha, & i = warp \\ F_{Si} \sin \alpha, & i = weft \end{cases}
$$
 (10)

During the tensile test of the fabric sample, because of the elongation the orientation of yarns changes causing shear deformation (Figure 6.a). At the same time a rather considerable crosswise contraction comes into being that increases the orientation change and the shear deformation as well as it locally increases the adhesion between the yarns as well.

Figure 6 Shearing deformation connected to the change in orientation angle of yarns at cutting angle 45° (a) and the correction model (b)

These effects can be taken into account in the FBC models at calculating the deformation of the yarns however in the case of the instantaneous failure the formula according to Equation (9) does not calculate with deformation therefore it is necessary to use some approximation. The changing orientation angle in the domain of α along the specimen (0≤X=x/L_o≤1) can be decribed as follows (Figure 6.b.):

$$
\alpha^*(X) = \begin{cases}\n\alpha - 90q_A \left[\left(\frac{4\alpha}{90} \right) \left(1 - \frac{\alpha}{90} \right) \right]^{n_A} \frac{(4X(1-X))^{n_L}}{1 + B_o/L_o}, & \text{warp} \\
\alpha + 90q_A \left[\left(\frac{4\alpha}{90} \right) \left(1 - \frac{\alpha}{90} \right) \right]^{n_A} \frac{(4X(1-X))^{n_L}}{1 + B_o/L_o}, & \text{weft}\n\end{cases}\n\tag{11}
$$

where q_A is proportional to the maximum increase of the orientation (Q_A) in the middle cross section (X=1/2) for warp and weft yarns. Exponents n_A and n_L determine the measure of

locality in the range of cutting angle and along the specimen. For inhomogeneous adhesion a similar expression can be used $(Y_s = l_s/L_o)$:

$$
Y_S^*(X) = Y_S \left[1 - q_S \left[\left(\frac{4\alpha}{90} \right) \left(1 - \frac{\alpha}{90} \right) \right]^{n_S} \frac{\left(4X(1-X) \right)^{n_L}}{1 + B_o / L_o} \right] \qquad Q_S = \frac{q_S}{1 + B_o / L_o} \tag{12}
$$

Material and Tests

The fibrous material used for testing and modelling was a plain woven fabric made of special false twisted multifilament PET yarns (Table 1). In order to provide proper basis for modelling and analysis tensile tests were carried out on yarns of 50 mm gauge length as well as on fabric samples of 50 mm width and 100 mm length cut out in directions 0 (warp), 15, 30, 45, 60, 75, and 90 (weft) degrees. The yarn samples were taken out of the fabric. The instrument applied was a Zwick Z005 universal tensile tester. In every case the rate of elongation was 100 mm/min and the number of tensile testing measurements was 10 or 3 in the cases of the yarns or the fabric, respectively.

Table 1 Nominal data of the fabric and its yarns

In order to characterize the interaction between the warp and weft yarns in main directions 3 pulling out tests were carried out where the length of the yarn gripped in fabric sample of 150 mm width was 120 mm (Figure 7).

Figure 7 Method of pulling out test

Results of Measurements

According to the results of tensile testing the tensile strength properties approximately are the same in spite of the different yarn densities in the fabric (Table 2) although the standard deviations are larger in warp direction (Figure 8). In the diagrams in Figure 8 the mean forceelongation curves shows the E-bundle-like behaviour of the yarns.

Table 2 Results of tensile measurements performed on yarns taken out of the fabric

Figure 8 Tensile test curves of yarns and its mean obtained by averaging point by point

Tensile test results show that the fabric in warp direction somewhat stronger than in the weft one, as well as the tensile strength strongly depends on the cutting angle and two local minimums and a maximum can be found in directions 15°, 75°, and 45°, respectively (Table 3).

Table 3 Breaking force results of tensile test of fabric specimens cut out different directions (warp: 0 degree, weft: 90 degree)

The tensile force versus displacement relation obtained during the pulling out test of warp yarn can be seen in Figure 9.a. The slippage steps occurred at displacements belonging to force peaks. Therefore the resistance against slippage can be characterized by the maximum values on the downset parts of the record which are included by the envelope (red curve) obtained by maximum smoothing (using a moving window similar to the moving averaging).

Figure 9 Record of pulling out measurement and its maximum smoothing (a), linear trend of the maximum values (b)

In accordance with the linear relationship, Equation (8) , the specific resistance force, f_b , that as a characteristic of the adhesion between the warp and weft yarns is related to the shearing effects as well, can be determined from the slope of the linear trend fitting (Figure 9.b). The results are contained by Table 4.

	Warp	Weft
$Av(f_b)$ [N/mm]	0,092	0,074
$SD(f_b)$ [N/mm]	0,005	0,003

Table 4 Results of pulling out measurements

Evaluation and Discussion

According to the concept of modelling the tensile behaviour of the fabric tested the first step was to create model warp and weft yarns for the FBC model calculations on the basis of the tensile test results of fabric samples cut out in the main directions. The mean forceelongation records obtained by averaging point by point were approximated by the expected tensile force response of nonlinear ET-bundle fitted by FibreSpace modelling program using the method of least squares (Figure 10).

Figure 10 Measured mean tensile force-elongation curves of fabric specimens cut in main directions related to one yarn and their non-linear E-bundle model

The parameters of the tensile characteristic and the breaking elongation of the model yarn determined from the fitting are summarized in Table 5 where the peak values of the model process can be compared to the measured strength properties related to one yarn. The critical adhesion length (l_s) and the critical length (l_{crit}) of the model yarns were calculated with Equation (3) from the mean breaking force of the model yarn $(Av(F_{\gamma s}))$ and the specific adhesion force (f_b) (Table 6).

Based on the simplified model introduced in the previous chapter the cutting angle and cross section dependent distribution functions of the active beard length for the 1', 1", and 0 gripped warp and weft yarns were determined similarly as the yarn number fractions were calculated (like those in Figure 5). Using that and the measured critical adhesion length values and the parameters of the model yarns (F_s) the mean tensile force of the fabric was computed with Equation (9) as a function of the cutting angle (α) and the cross section coordinate (x).

Fabric specimen		Property	Cutting angle	
			0 degree	90 degree
Tensile test Measured		Av(Ff,peak) [N]	10,5	9,5
		$Av(\lambda f, peak)$ [mm]	30,9	23,1
Tensile characteristic Model Breaking yarn elongation		c [N/mm]	0,60	0,65
	λ 0 [mm]	12,0	7,5	
	λ 1 [mm]	6,0	4,0	
		$Av(\lambda yS)$ [mm]	32,0	24,8
		$Av(\lambda yS)-u1$ [mm]	26,0	20,8
		$SD(\lambda yS)$ [mm]	1,1	1,2
	Breaking force	$Av(FyS)$ [N]	12,0	11,2
Model	Peak values	Av(Ff,peak) [N]	10,2	9,5
fabric		$Av(\lambda f, peak)$ [mm]	28,8	22,6
Comparison of model and		Relative square	2,96	1,87
measurement		error $[\%]$		

Table 5 Measured and modelled properties of fabric specimens cut in main directions related to one yarn

	Warp	Weft
$Av(F_{\nu S})$ [N]	12,0	11,2
$l_{\rm s}$ [mm]	130	152
$l_{\rm crit}$ [mm]	261	305

Table 6 Critical lengths of the model yarns

Figure 11 shows two results of calculating the mean breaking force of the fabric sample along its length at cutting angles 30° and 45° (without any orientation or adhesion correction) that well demonstrate that the critical cross section of minimum tensile strength can be found in the middle of fabric sample.

Figure 11 Normalized breaking force along the specimen at cutting angles α =30 $^{\circ}$ (a) and α =45^o (b)

Therefore for further calculations the place of the critical cross section was fixed $(X=x/L_o=1/2)$ and the the mean breaking force of the fabric sample was computed at different cutting angles. According to the results obtained without any orientation or adhesion correction (Figure 12.a) the tensile strength rapidly decreases on going from the main directions (0°, 90°) to 45° and comparing to the measured average values, these decreases are too large. Applying the correction formulae of the orientation angle and the critical adhesion length

good fitting could be gained in the range (30°, 60°) however considerable difference remained at 15° and 75°. This latter may partly be explained by gripping problems and the observed draping however its clarification needs further investigation.

Figure 12 Measured and modelled values of the normalized mean breaking force versus cutting angle (α) without (a) and with (b) correction

The parameter values used for correction are summarized in Table 7 whereas the maximum corrections were about 27% for both the yarn orientation angle and the critical adhesion length.

	Warp	Weft
l s [mm]	130	152
q_A [-]	0,41	0,41
n_A [-]		10
$Q_A[\%]$	27,3	27,3
q_S [-]	0,4	0,4
n_S [-]	5	5
Q_S [%]	26,7	26,7
n _L [-]		

Table 7 Model parameters for the orientation and adhesion corrections

Summary

The model yarns determined by nonlinear E-bundles and the simplified FBC model of the fabric samples based on tensile test and yarn pulling out measurements and the relationships of the yarn number fraction and the active beard length made it possible to study and analyse the effects of the cutting angle and the place of the critical cross section. In the case of instantaneous failure mode the simplified FBC model without any structural correction strongly underrated the mean breaking force for the cutting angles between 15° and 75° while this model with correcting the yarn orientation and the shearing effects related adhesion provided good fitting between 30° and 60°. These results give an important contribution to realizing the correct FBC models of the fabric samples in the FibreSpace program, in which the deformation, crosswise contraction, damage, and failure as time dependent processes are included.

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