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Application of the Tsai's Modulus and Double-Double concepts to the Definition of a New Affordable Design Approach for Composite Laminates.

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Abstract

Composites design has always been perceived to be complicated, time-consuming and therefore not affordable. With a paradigm shift, it is possible to make composites as straightforward as metals. Each metal is characterized by two key material properties: its Young's modulus and strength. The adoption of the Tsai's modulus as the only stiffness constant characterising any CFRP, and failure strain as the only measure of strength, we can make composite design as straightforward as metals design. Actually, with these two material characteristics, for any CFRP, material, laminate, and strain allowable can all be scaled, just like metals. Another break-through is the replacement of the traditional quad laminates of 0, ± 45 , 90 by double-double (DD) in

$[\pm\Phi/\pm\Psi]$. This replacement allows to substitute a discrete collection by a continuous field of orientations. A few dozen choices, replaced by hundreds. Indeed, double-double can make structures lighter than quad cannot do, thanks to the Lam-search optimizer, which is able, in combination with Fem tools, to sort, instantaneously, the best DD angles allowing laminates stiffness and strength tailored on specific loading conditions. The bottom line: Tsai's modulus and double-double are new but are simple conceptually, and Lam-search makes their use practical. In the end, lower weight and lower cost are derived by DD to deliver aggressive taper and 1-axis layup that will be stiff, strong, easy to layup, not prone to error, wrinkle, warpage and delamination.

Keywords: Layered structures, Carbon fibre, Mechanical properties, Laminate mechanics, Layup design, Double-Double laminate.

1. Introduction

Since the 1960s when the use of composites for airplanes started, the quad laminates of 0° , $\pm 45^\circ$, 90° plies have been the standard, with mid-plane symmetry and 10 percent rules, and have not been changed [1–3]. Although numerous advancements have been introduced and are continuously being introduced to composites and their layup methods (e.g. for ductility [4, 5], coupling behavior [6, 7] or thickness and layup optimization techniques [8,9]), the industry-standard layup method remained practically unchanged. The main reason for not changing the quad and the conventional guidelines is convenience. It is hard to change unless there is a good reason. Now there is a good reason. Design with metals is easy because they, actually, have only one stiffness and one strength parameter. With composites, we now can have the same one stiffness parameter in Tsai's modulus

[10-14], and the same one strength in designated failure strain allowable; hence, the design process between metals and composites becomes the same. The quad laminates are too difficult to design and manufacture because the building block sub-laminate is too thick. A better family of laminate can be identified in double-double [15] which opens up opportunities for simpler and lighter laminates hardly possible with the quad. As an example, a hard laminate in $[0_5/\pm 45_2/90]_S$ and a double-double in $[\pm 0/\pm 50]_{5T}$ have the same stiffness, strength in smooth, OHT (open hole tension) and OHC (open hole compression). It is possible to demonstrate that only double-double can make the structure lighter, with a faster layup at lower cost. Finally, Lam search, with its batch process is able to identify, through ranking, the best laminates robust, reliable and automated.

This paper is aimed to demonstrate that, with Tsai's modulus and double-double (DD) laminates of $[\pm\Phi/\pm\Psi]$, it is possible to find much simpler methods for design and manufacturing leading to significant weight savings hardly possible with the traditional quad [16, 17]. Instead of 4 required elastic constants for plies and resulting laminates, only Tsai's modulus will be sufficient for all CFRPs. Double-double sub-laminates require only 4 plies, instead of up to 20 for the quad. Instead of thousands of stacking sequence permutations, DD has only 4 permutations. Optimization is not only possible but straightforward. Thinner building block reduces minimum gage, and can be homogenized where mid-plane symmetry is naturally satisfied [18]. Aggressive ply drop in DD laminates, one at a time, not two for symmetry, can reduce weight more effectively than with the quad laminates. Layup speed is higher, scrap rate lower, and laminates with taper can be manufactured to reflect the intended design features. Tsai's modulus and double-double are more than novel. They can make composites easier to use, more competitive and can realize many applications not considered hitherto. While quad and DD may have comparable strength, there are

significant differences in how tapering can save weight, flexibility in layup can save time and cost in manufacturing, and other notable features all in favor of DD.

This paper aims to discuss the idea, advantages and validity of both the Tsai’s modulus and the double-double theories. Furthermore, it will be shown that double-double laminates are worthy successors of quads and that , with Lam search in combination with FEM tools, they are ready to be used in the industry.

2. Tsai’s modulus

Tsai’s modulus is the sum of the diagonal components of a matrix; i.e., $Q_{11} + Q_{22} + 2Q_{66}$. This invariant is derived from the transformation of the stiffness matrix [Q]. It is mathematically based, and exact [14].

2.1. Tsai’s modulus invariance

It becomes a material property with a less than 2 percent variation among nearly all carbon fiber reinforced polymer (CFRP). Such small variations are seen in Figure 1 for a Tsai’s modulus normalized lamina stiffness matrix [Q]* (based on [19]).

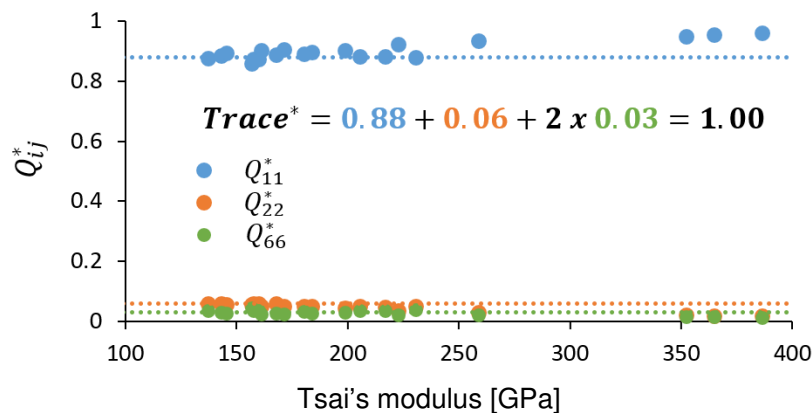


Figure 1 Average Tsai’s modulus-normalized stiffness [Q] (based on [19])

It is sufficient to have Tsai's modulus as the one and only one constant needed for design. As shown in Figure 2, it can be calculated from the lamina longitudinal modulus E_1 of any CFRP by normalizing by 0.88. Or it can be calculated from the Young's modulus of any laminate (e.g., for [0/90] the Tsai's modulus could be evaluated from E_x by normalizing by 0.47.

It should be recognized that the transverse and shear moduli of [0] are so small as compared with the longitudinal moduli, their average values of 6 and 3 percent, respectively, would be good enough for calculating the stiffness of laminates. They need not be measured for preliminary design of laminated structures. The shear test in particular is so difficult and so unsure of the data. It is probably just as safe for design to take its value as 3 percent of Tsai's modulus.

It is along the same thinking that design can be based on representative data rather than B-allowable. If we have trust in E_1 being 88 percent of Tsai's modulus, and E_1 can be evaluated by the rule-of-mixture starting from the longitudinal young modulus of the fibres E_f , then Tsai's modulus could be simply evaluated, as a first approximation, by normalizing the longitudinal young modulus of the fibres (multiplied by the fiber volum fraction v_f) by 0,88. This estimate without testing may be sufficient for many preliminary designs. Working with composites will be more popular if we make the process simpler. Believing in Tsai's modulus can be the first step.

2.2. Laminate stiffness from Tsai's modulus

Laminate stiffness in absolute and Tsai's modulus-normalized values are shown below for six materials: IM7/977, IM7/8552, T8S/3900, T800/Cyt, T7 C-Ply 64, and T650/Epoxy (Figure 2). Each material has 3 stacking sequences: [Quasi-isotropic], and hard in $[0_5/\pm 45_2/90]$ at 0° and 30° orientations. Note that Tsai's modulus is unique for each material, and is the same for all 6 laminates, at 218, 192, 168, 182, 163 and 161, respectively. For the same laminate, their Tsai's modulus-normalized stiffness values are the same for all CFRP; i.e., for QI, $A_{11}^* = 0.37$ for both

all the laminates. In fact, this laminate factor (lam^*) applies to all CFRP, not just these six materials.

IM7/977									IM7/8552																										
QI			Tsai's			Hard 0°			Tsai's			Hard 30°			Tsai's																				
81	24	0	81			121	20	0	121			97	26	19	97			71	22	0	71			108	18	0	108			86	24	17	86		
	81	0		81			48	0		48			60	12		60			71	0		71			43	0		43			53	11		53	
		28		57				24		48				30		61				25		49				21		41				26		53	
0,37	0,11	0,00	0,37			0,56	0,09	0,00	0,56			0,44	0,12	0,09	0,44			0,37	0,12	0,00	0,37			0,56	0,10	0,00	0,56			0,45	0,13	0,09	0,45		
	0,37	0,00		0,37			0,22	0,00		0,22			0,28	0,06		0,28			0,37	0,00		0,37			0,22	0,00		0,22			0,28	0,06		0,28	
		0,13		0,26				0,11		0,22				0,14		0,28				0,13		0,26				0,11		0,22				0,14		0,28	
		1,00						1,00						1,00						1,00						1,00						1,00			

T8S/3900									T800/Cyt																										
QI			Tsai's			Hard 0°			Tsai's			Hard 30°			Tsai's																				
63	20	0	63			95	17	0	95			75	22	15	75			68	22	0	68			103	18	0	103			82	24	17	82		
	63	0		63			38	0		38			47	9,4		47			68	0		68			41	0		41			51	10		51	
		21		43				18		36				23		46				23		46				20		39				25		50	
0,37	0,12	0,00	0,37			0,56	0,10	0,00	0,56			0,45	0,13	0,09	0,45			0,37	0,12	0,00	0,37			0,56	0,10	0,00	0,56			0,45	0,13	0,09	0,45		
	0,37	0,00		0,37			0,22	0,00		0,22			0,28	0,06		0,28			0,37	0,00		0,37			0,22	0,00		0,22			0,28	0,06		0,28	
		0,13		0,25				0,11		0,21				0,14		0,27				0,13		0,25				0,11		0,21				0,14		0,27	
		1,00						1,00						1,00						1,00						1,00						1,00			

C-Ply 64									T650/epoxy																										
QI			Tsai's			Hard 0°			Tsai's			Hard 30°			Tsai's																				
60	18	0	60			90	15	0	90			72	20	14	72			60	18	0	60			89	15	0	89			71	20	14	71		
	60	0		60			37	0		37			46	8,8		46			60	0		60			37	0		37			45	8,7		45	
		21		42				18		36				23		45				21		41				18		35				22		44	
0,37	0,11	0,00	0,37			0,55	0,09	0,00	0,55			0,44	0,12	0,09	0,44			0,37	0,11	0,00	0,37			0,55	0,09	0,00	0,55			0,44	0,12	0,09	0,44		
	0,37	0,00		0,37			0,23	0,00		0,23			0,28	0,05		0,28			0,37	0,00		0,37			0,23	0,00		0,23			0,28	0,05		0,28	
		0,13		0,26				0,11		0,22				0,14		0,28				0,13		0,26				0,11		0,22				0,14		0,28	
		1,00						1,00						1,00						1,00						1,00						1,00			

Figure 2 Absolute and normalized [A] of six CFRP

The diagonal components of the laminate stiffness are defined by the partitioning of Tsai's modulus; e.g., for QI the partitioning is 37-37-26 percent; for hard, 56-22-22 percent. (Shear modulus must be doubled.) Laminate stiffness is therefore the product of Tsai's modulus and the partitioning as described. Tsai's modulus is a material property, and partitioning is geometric, dimensionless, and must add up to 100 percent. This is why Tsai's modulus is the one and only one constant needed for design. It gives the scale or magnitude for laminate factor for all laminates. No other constant can be so universal.

3. Quad versus DD

3.1. Quad weaknesses

The 20th Century laminate in quad is built on an additive process. It is a collection of the properties of 0, ±45, and 90 plies. In Figure 3, a carpet plot [20] representing feasible quad laminates is presented. To the right of Figure 3 is the field for Tsai’s modulus-normalized A_{11} as functions of percentages of [±45] versus [0]. Only 16 discrete laminates (not 66), shown in brown boxes make the building block sub-laminate to be 10-ply thick. Percent [±45] must be 20, 40, 60, and 80 percent because they come in pairs. Cannot be 30 percent unless the sub-laminate is 20-ply thick.

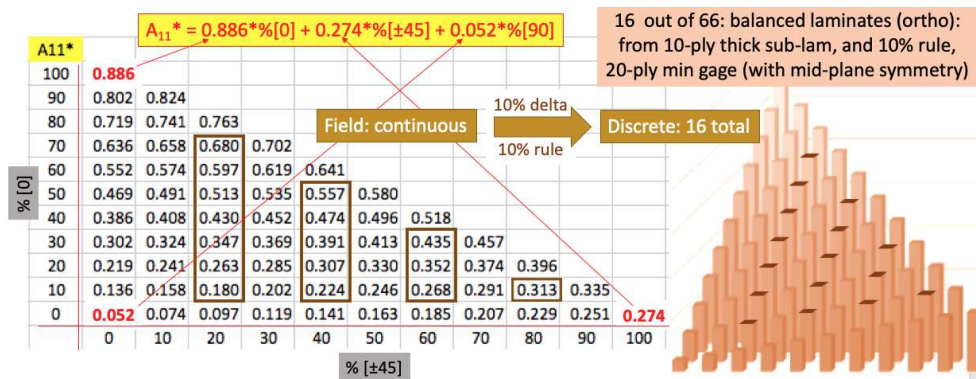


Figure 3 How discrete quad laminates are assembled and shown in a “carpet plot”

Similarly, for nine 8-ply thick sub-laminates the ply percentages must change from 10 percent to fraction increments of 1/8 or 12.5 percent. For four 6-ply sub-laminates the ply percentages are fraction increments of 1/6 or 17%. So we have a total of 16 + 9 + 4 = 29 sub-laminates. For quad the choice of sub-laminates is limited to 29 (no limit on double-double), and being discrete they are no connections among them. Laminate property makes a discrete jump if one ply is added or subtracted from a laminate or sub-laminate. There can be no interpolation; i.e., either 4 or 5 [0], not 4-1/2 [0] (DD can go as smoothly as desired like 30 to 35, or 30 to 31.6 degree).

This is just the first limitation of quad. There are many more related to their discrete nature and to their fixed 0°, 90°, +45°, -45° angles.

3.2. Stacking sequence permutations

As a minimum for the quad, there are 4 plies, one in each of 4 ply angles. That collection gives quasi-isotropic stiffness. If orthotropy is needed, plies of selected angles must be added (not traded). So the thickness of the laminate would increase from 4 to 5, ... to 10. In fact, it is assumed to use these laminates as sub-laminates to be repeated like building blocks in their stacking to form a thicker, desired laminate. Inherent in this quad laminate is the need to have mid-plane symmetry to prevent warpage from cooldown. Then the sub-laminate must come in pairs and would be minimum 12-, 16-, and 20-ply thick.

The number of permutations of stacking sequence for quad will increase with the number of plies in the sub-laminate. In Figure 4, it starts with 24 for 4-ply and 25,250 for 10-ply sub-laminate (DD remains at 4 because it has two each $\pm\Phi$ and $\pm\Psi$). Optimization is often not possible for quad because there are too many stacking sequences. It becomes worse to have different laminate in each element. Blending these laminates has emerged as a specialized but absolutely subjective art (no blending at all for one double-double if it is for the entire component).

Quad*		DD
$[\pi/4]$	$[0_p/\pm 45_q/90_r], 1 \leq [p,q,r] \leq 3$ 4-ply sub-lam [1,2,1] 	Permu- tations 4
[Hard2]	5-ply sub-lam [2,2,1], [1,2,2] 	Permu- tations 4
[Others]	6-ply sub-laminate → 180 8-ply sub-laminate → 2,520 10-ply sub-laminate → 25,250	Permu- tations 4 4 4

* Every quad design will be history making: no chance to be repeated; cannot optimize; cannot scale

Figure 4 Comparisons between permutations of stacking sequence for quad and DD

3.3. Quad design examples

A design that looks like a stack of bricks is a consequence of using quad, is seen in Figure 5. There are thousands of discontinuities. As we will see that double-double can eliminate all internal discontinuities because the building block is always 4-ply thick, or 2-ply thick if thin plies are used. The difference in ply drop method is totally different between quad and DD.

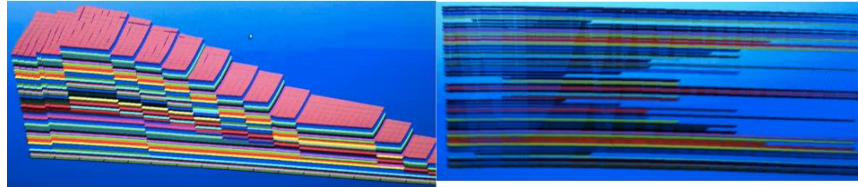


Figure 5 Not uncommon consequences quad-built laminated component: so heterogeneous

Ply drop is unique with composites. It is one of the best ways to reduce weight. With quad, ply drops must come in pairs in order to maintain mid-plane symmetry. The laminated component would warp upon cooldown if there is no symmetry. Such ply drops will change laminate properties for quad design, but not for double-double because it is homogenized by design.

Working with quad, ply drop is coupled with blending. It is a nightmare for both design and manufacturing. Compromise is necessary but often times manufacturing is forced to do what it can and cannot live with the design features. The resulting ply drop profile is not optimum to deliver the desired strength or toughness requirement. Double-double has thinner sub-laminates (as low as 2 versus as high as 20 for quad), and its laminate is readily homogenized. Then it is naturally symmetric and ply drop can be one at a time. As we will show that the drop can be placed in any position. That makes tapering much simpler to design and more producible. This is another case where DD can do but quad cannot.

3.4. Layup options

Layup speed of unitape is the fastest when the orientation is along the 0-axis assuming that the component is rectangular. The off-axis plies like +/- 45 are the slowest because the layup machine

slows down to near zero speed as the tape laying approaches one of the corners of the component. One way to avoid the off-axis ply is to have multiple angles pre-plyed in along the axis of tape or fabric. There are such bi- and tri-axial tape and fabric but they are often limited to $[0/\pm 45]$ and $[0/\pm 60]$. Double-double non-crimp fabric (NCF) can be made with wide-ranging angles, say, a single-double in $[\Phi/\Psi]$ and its companion single-double in $[-\Phi/-\Psi]$. When the former is placed over the latter, a double-double is formed in $[\Phi/\Psi/-\Phi/-\Psi]$. If the initial single double in $[\Phi/\Psi]$ is flipped over, it will have $[-\Psi/-\Phi]$. Then the stacked laminate will be $[\Phi/\Psi/-\Psi/-\Phi]$ which is essentially $[\pm\Phi/\pm\Psi]$ when multiple NCF are stacked. Since there are only 4 ply angles in double-double, their stacking sequences are limited, and shows no significant difference in properties among the 4 different stacking sequences. Such flexibility in double-double does not exist in quad because the number of permutations in stacking is huge. It will take five or more repeats to become homogenized that means 100 plies or 12.5 mm thick for a specific quad. Many structures do not need such thick laminates. On the other hand, many structures have much lower minimum gage requirement. Twenty-ply thick quad is too thick to meet this lower limit requirement. Double-double is a much more efficient laminate. It can reach a homogenized laminate many times faster than quad (1/10 the number of plies, as we will see), and have much lower minimum gage (the same 1/10) to meet the low limit of industrial applications that the quad cannot compete.

3.5. Ply dispersion

Double-double is based totally on the field theory. Direct field-to-field comparisons between quad and DD are seen in Figure 6. Being a collection of 4 ply angles, quad cannot sustain continuity and is forced to go to 29 discrete laminates in 6-, 8- and 10-ply building blocks. DD, on the other hand, stays continuous and has no restriction on the value in ply angles. Can meet the demand by zooming the ply angle increments to whatever values other than 7.5 degree shown on the left of Figure 6. Every laminate can have a slope (first derivative) that makes optimization possible. Being

discrete, quad cannot be differentiated; i.e., no derivatives to help optimization or a simple algebraic operation.

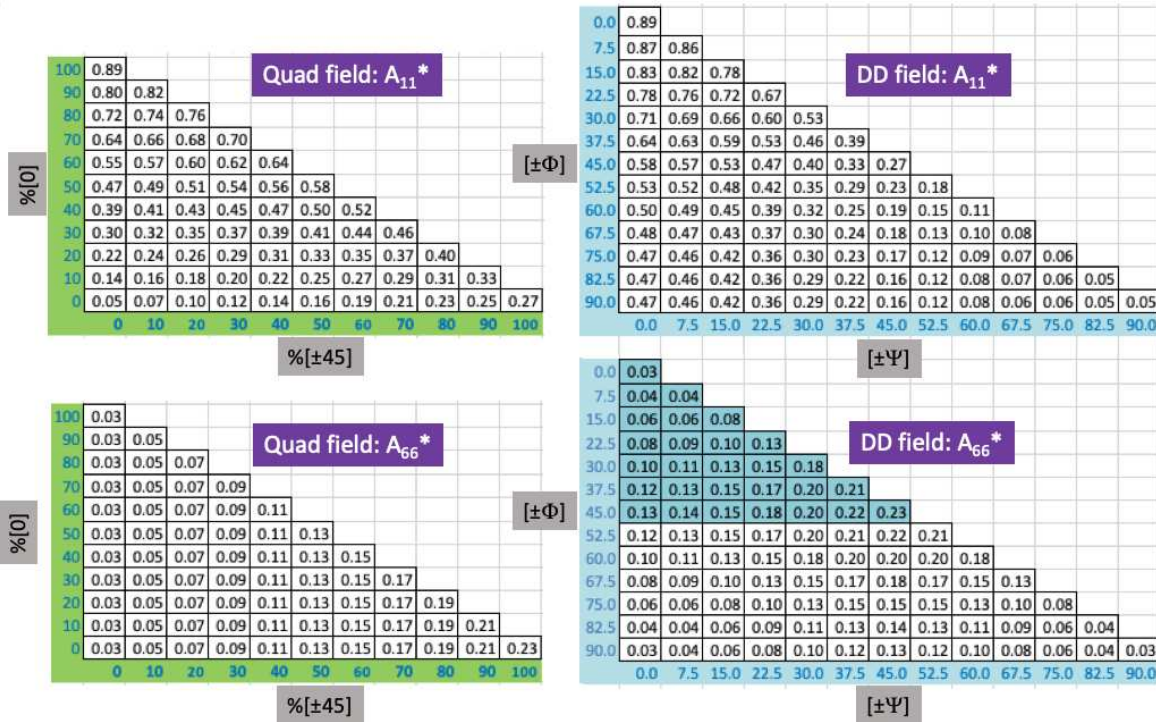


Figure 6 Quad versus DD in terms of field theory but quad must go discrete, no continuity

These are profound differences between quad and DD mathematically. For everyday use, the difference between them is just as dramatic. As mentioned earlier, quad with 6-, 8-, and 10-ply thick sub-laminates must be symmetric. That makes their minimum laminate thickness to 12-, 16-, and 20-ply thick. Double-double is 4-ply thick, and can also be 2-ply thick if thin plies (75 gsm) are used. Thus a 20/2 or 10/1 ratio in their relative thickness is there between quad and DD. This difference is reflected in how fast homogenization can be realized. Plies are finely or uniformly dispersed. Once this dispersion and homogenization is achieved (as we shall see that for DD it is 32 plies or 8 repeats) the need of mid-plane symmetric and ply drops in pairs will no longer need to be enforced. Quad can also achieve homogenization although with many more plies; there is no advantage for quad to make ply drop any simpler for manufacturing advantage.

3.6. Homogenization

As we have claimed, a homogenized or uniformly dispersed laminate is an important attribute for double-double. It is reached mathematically when $[A^*] = [D^*]$ and $[B] = 0$. (We arbitrarily consider the largest component of $[B^*]$ to be 0.02 to be sufficiently small to cause warpage in cooldown from cure temperature.) When a sub-laminate is often made thin like in DD, homogenization is more easily achieved, but not so for quad because its sub-laminates are thick. Uniform dispersion is still possible but would take many more plies. The difference between reaching a homogenized state by a quad in ‘hard laminate’ in $[0_5/\pm 45_2/90]_{r/s}$ and its DD replacement in $[\pm 0/\pm 50]_{r/t}$ is shown in Figure 7.

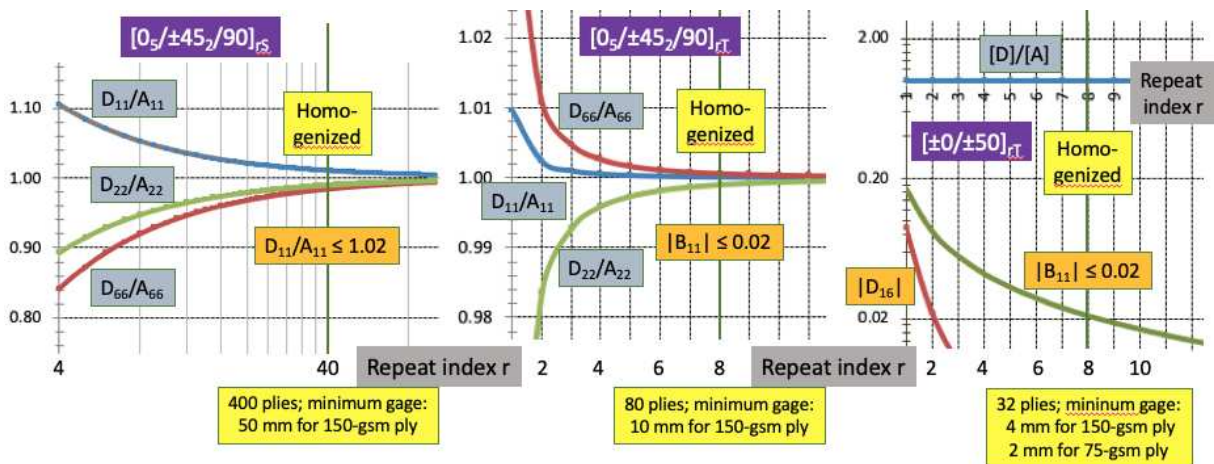


Figure 7 Homogenization: more than 400 plies needed for quad; only 32 for DD: 12x difference

Quad must go far beyond 10 repeats to meet the requirement of $[A^*] = [D^*]$ when the gap between the top and bottom of D_{11}^* merges into one line. In the meantime, the degree of homogenization of DD is controlled by the magnitude of $[B]$, as seen in the right of Figure 7. There is a more than 10-fold difference between the thickness of the quad and DD in reaching a state of homogenization. This difference becomes important in how laminates can be designed and manufactured. More will be said later.

For quad, mid-plane symmetry is required to prevent thermal warpage. The top half of the laminate is a mirror image of the bottom half. The minimum thickness of quad is doubled to 12-, 16- and 20-ply. If a laminate is heterogeneous as all quads are, ply drop will have to be in pairs and with mid-plane symmetry. Like keeping track of the mid-plane, ply drops in pairs are just as difficult to implement and prone to errors. DD, on the other hand, can have ply drop one at a time, positioned anywhere (interior body or exterior surfaces) and highly producible.

3.7. Single ply drop

Yes, single ply drop can be fast, less prone to error, and stronger and more resistant to edge and possibly general delamination. Examples are shown in Figure 8. They are clean, smooth, easy for layup, and with no random discontinuities; totally different from those shown in Figure 5 for quad. Again, DD being thin can provide taper to save weight that quad cannot do. Many fibers connected from root to tip. This is different from the examples given in Figure 5 where a wing structure looked like a stack of bricks, with no continuous flow of fibers from root to tip. We will show shortly with test data that ply drops on the exterior surfaces are strong and tough, and do not peel off prematurely.

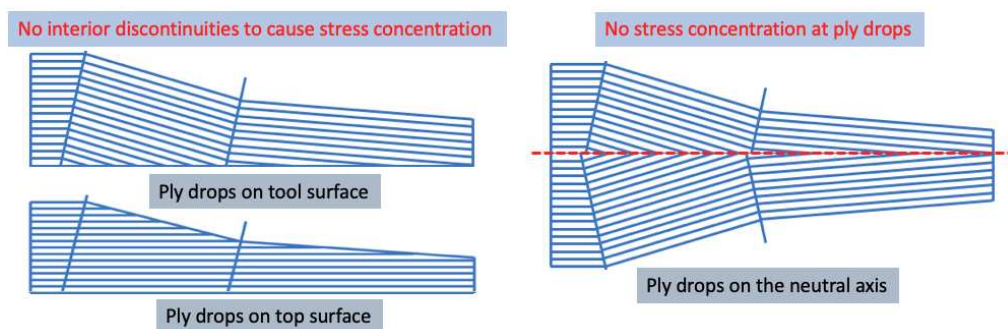


Figure 8 Single ply drop facilitated by DD can be positioned for fast layup and/or strength

Such beams have been made by Nashero (by Naresh) with C-Ply supplied by Chomarat (by Brian) and tested at NIAR at the University of Wichita State (by Waruna with his data shown below), and

the University of Porto (by Albertino). Remarkable static and fatigue strengths of ply drops on the exterior surface were found, as seen in Figure 9. University of Porto's data on static strength were consistent with those from NIAR.

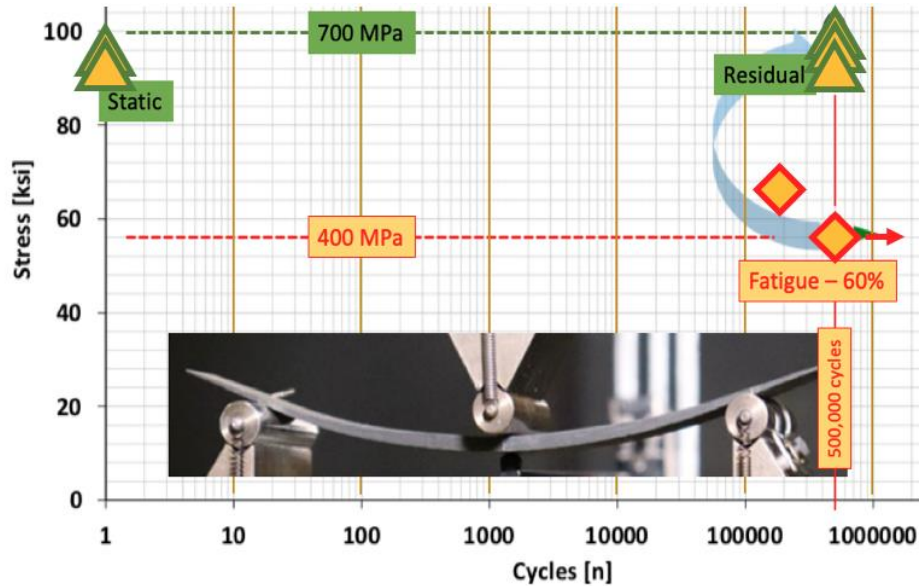


Figure 9 Three-point bend test of a dual-tapered beam with $[\pm 0/\pm 50]$ DD ply stacking

The simplicity of this sliding method cannot be overemphasized. The neat sliding of plies of the same size is the essence of this discovery. This is quite a difference if individual plies of varying sizes must be pre-cut and placed, or placed with a variable cut in the stacking by the traditional method. Such stacking is shown in the left of Figure 8.

4. Best Laminate and Thickness Optimization

Figure 10 depicts the methodology to integrate finite element (FE) simulation with Lam Search for a) determining best DD layup, and b) optimizing thickness profile of a composite laminate subjected to in-plane loads. In-plane strain distribution is obtained in the laminate for a given geometry, load, material properties, and boundary conditions using FE simulation. The strain distribution in global coordinates are averaged over 7×7 grid discretization of the laminate and

are thereafter used to obtain laminate stress distribution on individual 49 cells (7×7 grid). Lam Search determines the best DD layup and estimates strength ratio (R-values) for the individual 49 cells based on Tsai-Wu based first ply failure criteria using these laminate stresses. The distribution of R-values across 49 cells provide basis for thickness scaling. The optimization loop as shown in Figure 10 continues until the R-values across 49 cells converge to unity (or close to unity).

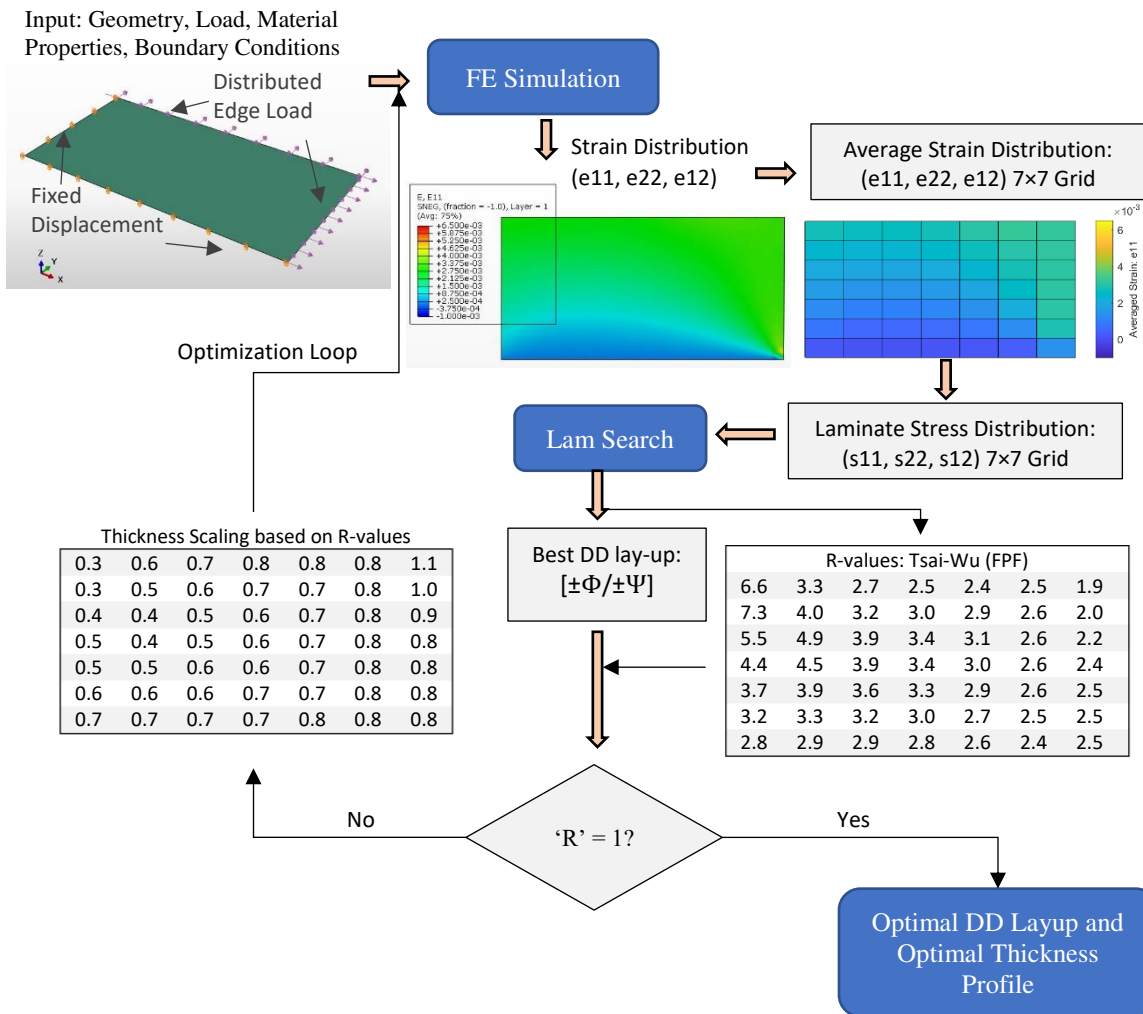


Figure 10 Integrated Lam Search and finite element based methodology for thickness optimization and best layup determination.

Figure 11 shows an example wherein a flat plate ($800\text{ mm} \times 400\text{ mm} \times 2\text{ mm}$), made up of quasi-isotropic (QI) laminate ($[0/90/\pm 45]_{sym}$) of uniform thickness is subjected to load ($N_x = 4N_y =$

375 N/mm, $N_{xy} = 75$ N/mm) distributed along the edges. Following the methodology presented in Figure 10, Lam Search is integrated with FE based optimization loop resulting in best DD layup for the given loading conditions. As can be seen from Figure 11, the best DD layup ($[\pm 22.5/\pm 52.5]$) resulted in optimized R-values close to unity for all individual 49 cells of 7×7 grid discretization of the laminate. The optimized thickness of the individual 49 cells is also shown in the figure resulting in considerable weight savings (67%) from the assumed initial conditions (2 mm uniform thickness).

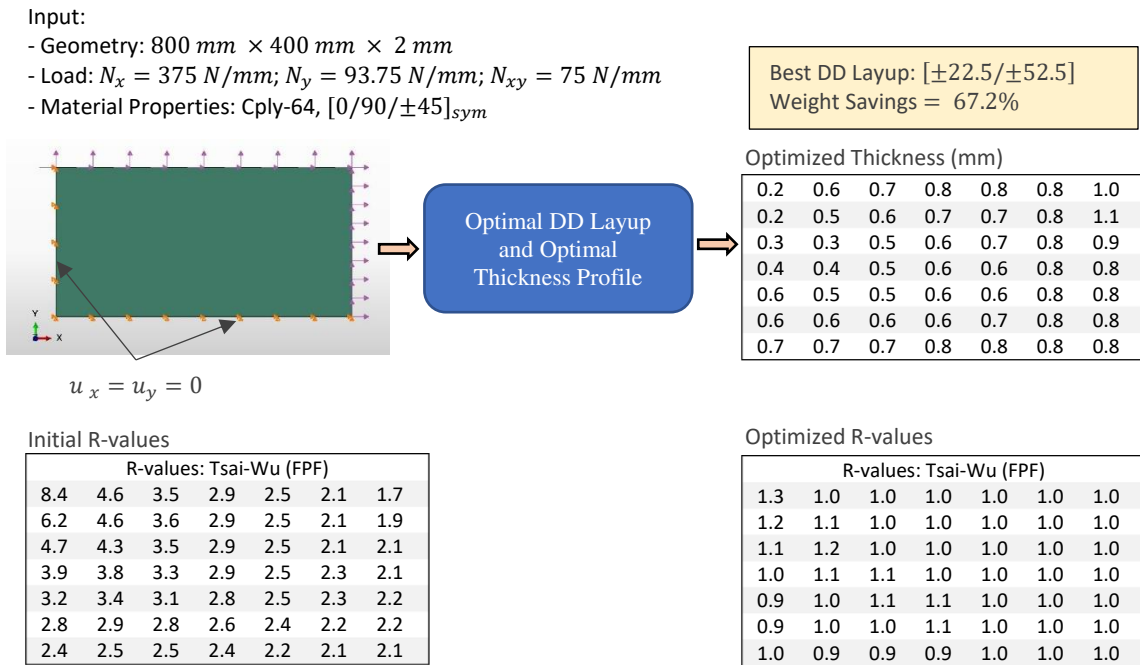


Figure 11 Thickness profile optimization of a flat plate CFRP laminate using 7x7 grid.

Table 1 shows the effect of different assumed initial layup configuration and laminate thickness on the optimized thickness profile and best DD layup estimated using Lam Search. The best DD layup for QI laminate is $[\pm 22.5/\pm 52.5]$, whereas the best DD layup for Square laminate ($[0/90]_{sym}$) is $[\pm 15/\pm 45]$ for the specified geometry and loading conditions. Even though the best DD layup for two different starting laminate sequence is not exactly the same but the difference in

average R-value is close to 1% as reported in Table 1, which indicates that for practical purposes the two best DD layup are almost similar for the specified geometry and loading conditions. It is also observed that different starting thicknesses do not change the outcome of best DD layup and optimized thickness profile. Thus, integration of FE based modelling and simulation with Lam search will be a highly effective tool to obtain strength-optimized composite laminate structures for specified loading scenarios.

Table 1 Effect of initial conditions on the estimation of best DD layup and optimized thickness profile using Lam Search.

Initial Starting Layup; and Initial Thickness	Initial Weight (kg) ($\rho = 1550 \text{ kg/m}^3$)	Best DD Layup	Average R-value (7×7 grid)	Optimized Weight (kg)	% Weight Savings
QI [0/90/ ± 45] _s ; 2 mm uniform thickness	0.992	[$\pm 22.5/\pm 52.5$]	1.02	0.325	67.2%
Square [0/90] _s ; 2 mm uniform thickness	0.992	[$\pm 15/\pm 45$]	1.01	0.305	69.3%
QI [0/90/ ± 45] _s ; 3 mm uniform thickness	1.488	[$\pm 22.5/\pm 52.5$]	1.02	0.325	78.2%
QI [0/90/ ± 45] _s ; 4 mm uniform thickness	1.984	[$\pm 22.5/\pm 52.5$]	1.02	0.325	83.6%

The optimized thickness profile estimated using Lam Search results in considerable weight savings but may not be easy to manufacture due to the high degree of mismatch in thicknesses of individual cells of the 7×7 grid. Figure 12 shows a simple tapering strategy wherein weight reduction is traded-off with ease of manufacturability by reducing the degree of mismatch in the thicknesses of individual cells. The revised thickness profile for the best DD layup would still result in 50% weight savings. A more advanced tapering strategy may be devised to result in even increased weight savings close to the optimal value.

Final Optimized Thickness (mm); Weight = 0.325 kg;
Weight Savings = 67.2%

0.2	0.6	0.7	0.8	0.8	0.8	1.0
0.2	0.5	0.6	0.7	0.7	0.8	1.1
0.3	0.3	0.5	0.6	0.7	0.8	0.9
0.4	0.4	0.5	0.6	0.6	0.8	0.8
0.6	0.5	0.5	0.6	0.6	0.8	0.8
0.6	0.6	0.6	0.6	0.7	0.8	0.8
0.7	0.7	0.7	0.8	0.8	0.8	0.8

Revised Thickness (mm); Weight = 0.496 kg;
Weight Savings = 50%

0.8	0.8	0.8	1.0	1.0	1.3	1.5
0.8	0.8	0.8	1.0	1.0	1.3	1.5
0.8	0.8	0.8	1.0	1.0	1.3	1.5
0.8	0.8	0.8	1.0	1.0	1.3	1.5
0.8	0.8	0.8	1.0	1.0	1.3	1.5
0.8	0.8	0.8	1.0	1.0	1.3	1.5
0.8	0.8	0.8	1.0	1.0	1.3	1.5

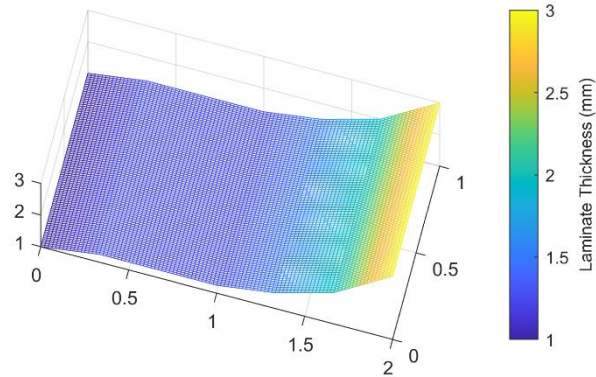


Figure 12 A simple subjective tapering strategy to trade-off weight reduction with ease of manufacturability.

In order to demonstrate the applicability of Double-double to the design of composite components with complex geometry, boundary and loading conditions, the same methodology described in Figure 10 has been applied to the skin of a stiffened composite panel of a fuselage barrel subjected to torsional and flexural loading conditions. As for the previous test case, the aim was to determine the best DD layup and the optimized thickness profile. In Figure 13 the geometry of the stiffened panel, the loading conditions and the subdivision of the skin in 42 cells are introduced.

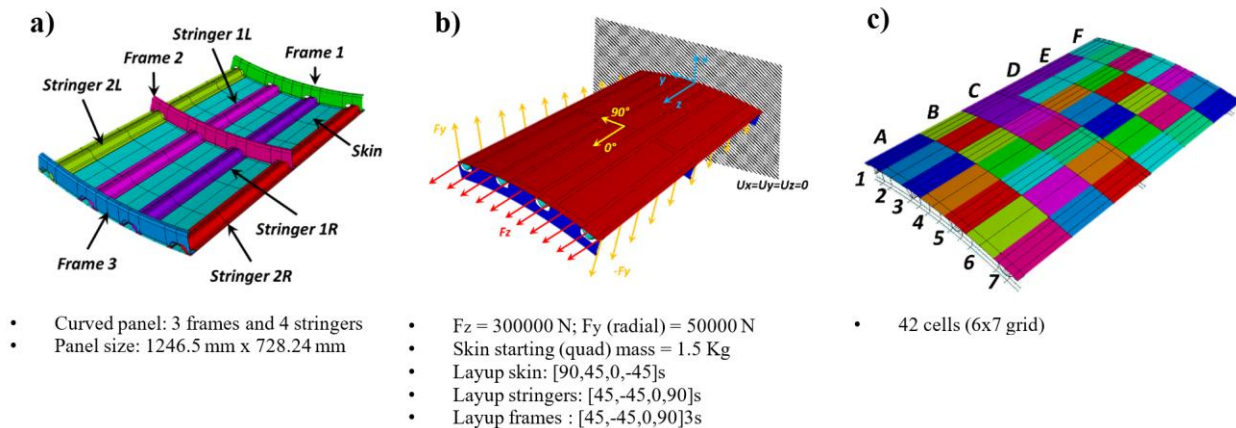


Figure 13 Curved stiffened panel test case: a) geometry; b) loading and boundary conditions; c) cells subdivision).

The optimized DD and thickness profiles have been obtained after 6 iterations between the Lam search and the FE simulations. As it can be appreciated in Figure 14, the iterations quickly led to a strong reduction of the average R in the cells and to the definition of stable DD angles.

The sparse thickness profile resulting from the application of Lam search, which is related to the mean global stress in the cells, has been modified (digitized) according to plies thickness and then tapered to increase the manufacturability by reducing angle variations along the hoop and axial directions. Contour plots of the resulting thickness profiles are introduced in Figure 14 for the analysed skin configurations. Results highlight, for the optimized, digitized and tapered configurations, an increase of skin thickness under the stringers' foot and a strong reduction in the bays. This result, emphasized for the digitized and the tapered configuration with constant axial thickness, was somehow expected due to the increase of stress and strains components at the stringers' foot. The introduced DD procedure removes layers, where possible, by reducing overdimensioning. Hence the weight reduction is expected to be more significant for those structural sub-components which less contribute to the load sharing.

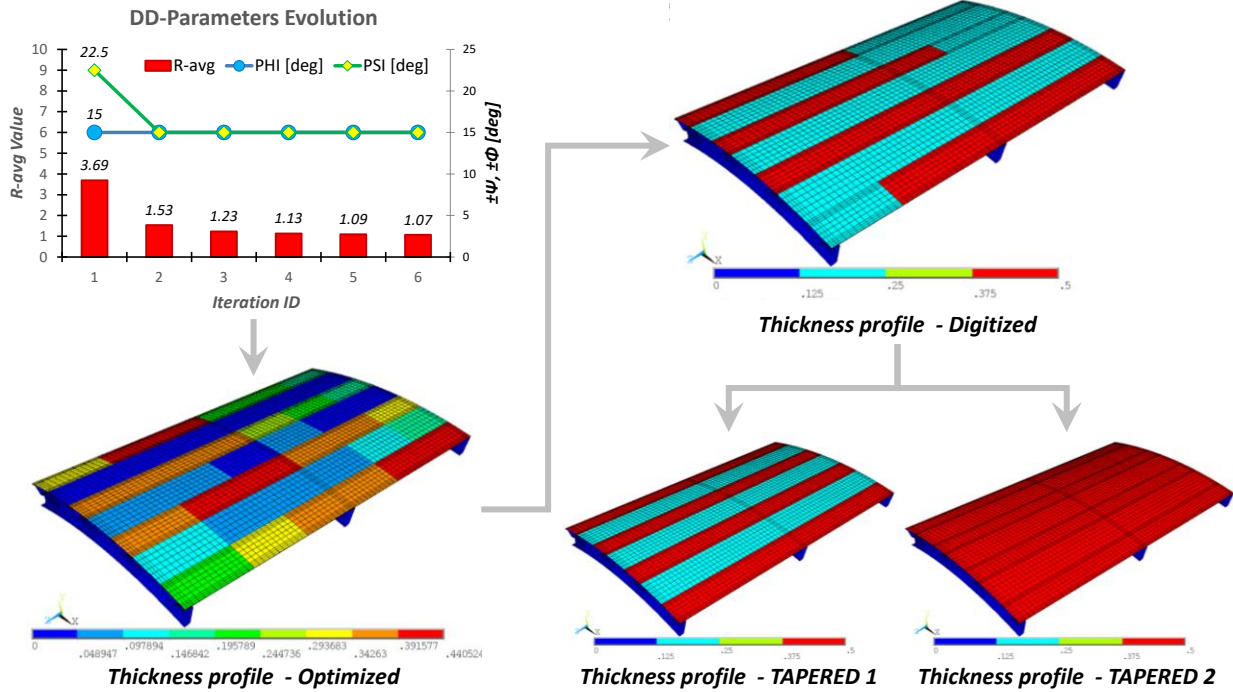


Figure 14 DD parameters evolution during iteration and thickness profiles

Table 2 shows the effects of digitizing and tapering operations on the skin thickness of the analysed stiffened composite panel and, consequently, on the resulting weight of the panel. The best DD layup for the skin laminate is $[\pm 15/\pm 15]$ for the specified geometry and loading conditions. As expected, the digitizing and the tapering operations which can guarantee a reduction of the manufacturing time and costs, lead to a reduction of the weight saving as well. However, as reported in Table 2, the DD skin configuration with a uniform thickness is still able to guarantee a weight saving of 51,6 percent, if compared to the starting quad configuration of the real fuselage skin panel. It is easy to imagine that, over the total fuselage skin area, the weight saving, induced by DD adoption, would be meaningful in terms of total Operating Empty Weight (OEW) reduction.

Table 2 Effect of digitizing and tapering operations on the thickness profile and weight of the panel skin.

	Initial Skin Weight (kg)	Final SkinWeight (kg)	% Skin Weight Savings

Optimised configuration	1.5	0.294	80.4%
Digitized configuration		0.507	66.2%
Tapered 1 configuration		0.555	63.0%
Tapered 2 configuration		0.726	51.6%

5. Toughness and fatigue behavior of DD laminates

Toughness is as important as strength. Based on limited available data, there is no evidence that DD is any weaker than a comparable quad. In Figure 13, open-hole tension (OHT) and compression after impact (CAI) were compared between quad in $[0/\pm 45/90]$ and DD in $[\pm 22.5/\pm 67.5]$. The laminate is T-700 C-Ply/M21, made in autoclave by Hexcel, Dublin, CA, and tested by Alan Nettles of NASA. While their uniaxial tensile strength is higher for quad than DD (with B-allowable at 600 over 510 MPa), their OHT and CAI were nearly identical. If these tests reflect laminate toughness, DD and quad with equal stiffness will have the same toughness.

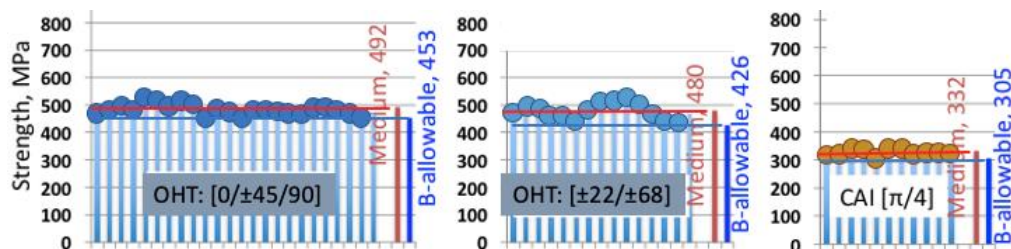


Figure 13 Comparison of OHT and CAI between $[0/\pm 45/90]$ and $[\pm 22.5/\pm 67.5]$

Additional data of uniaxial tensile and compressive strength, with and without a hole, and stiffness between ‘hard’ in $[0_5/\pm 45_2/90]_S$ and matching DD in $[\pm 0/\pm 50]_{5T}$ showed again nearly identical values as seen in each box in Figure 14. The data came from T700/epoxy laminates provided by Toray America, and testing by Waruna of NIAR.

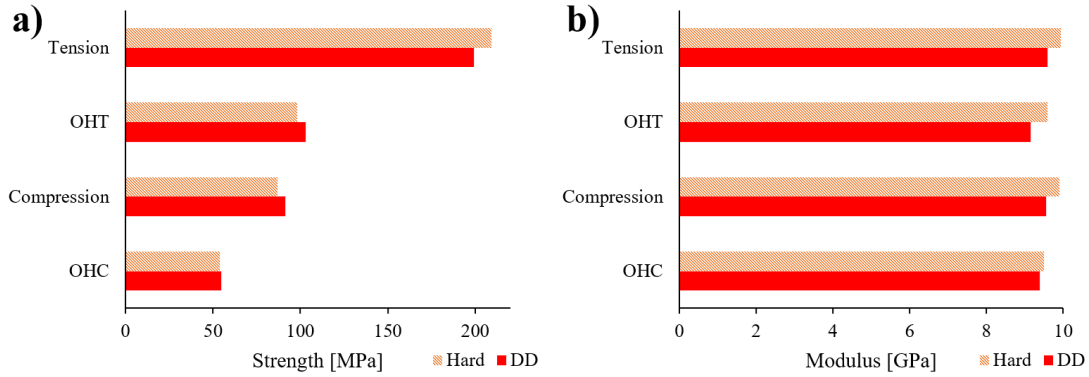


Figure 14 Comparison between DD and quad in a) uniaxial strength and b) stiffness. DD: $[\pm 0/\pm 50]_{5T}$, hard quad: $[0_5/\pm 45_2/90]_s$. OHT: open-hole tension, OHC: open-hole compression.

While more data will be needed as use of DD expands, there is no indication at this point that DD is not competitive with a comparable quad. However, the toughness performance of a composite laminates cannot be fully characterized via only the quasi-static strength but also the damage evolution characteristic should be taken into account. Actually, damage evolution in quad and DD laminates with the similar strength may be not the same or even significantly different, depending on ply angles and test methods. If both coupons have the same percentages of $[0]$ their strength properties have been found to be similar. We would expect higher fewer damage evolution from DD if ply drops are on the exterior surface thus freeing internal defects like resin rich area, fiber breaks and wrinkles that causes so many mechanisms of damage. DD also has thinner plies which should also suppress delamination; hence a much tougher laminate from DD over quad can be expected.

Based on data on a dual-tapered beam, with all ply drops on the exterior surfaces, subjected to static, fatigue and residual strength measurements, very respective strength values were found.

Furthermore, fatigue tests on $[\pm 0/\pm 25]$ and $[\pm 0/\pm 50]$ have been run. Both showed data of comparable quad coupons. Being homogeneous, DD is expected to be better than comparable heterogeneous quad. For DD, strength data including fatigue are expected to be the same between tension, compression and bending. Quad on the other hand can have significant difference among the same tests. While data for such comparison is not available at this time, a concern of DD for being weaker does not appear to be the case.

Finally, these differences in damage evolution and fatigue behaviour between quad and DD can also result in different strategies in damage monitor and structural repairment of a composite structure during its service life affecting Life Cycle costs. Hence, considerations on this costs related aspect should be included when comparing the Toughness and fatigue behaviour of Quad and DD composite components.

6. Lam search: an optimization tool for preliminary design

Lam search is a program that can find the best laminate for a set of multiple loads from which the controlling load, and the relative strength ratio (safety factor) of smaller loads in the same zone can be determined. The reduction of the thickness of all locations of the smaller loads will show the possible weight reduction of the zone in a component. It is recommended to consider one zone for each component. Started two years ago, Lam search grew from the MicMac that was driven by a macro called Chart-quick.

We were not aware of comparable tools that could find the best laminate like Lam search. It opened up the advantages of DD over quad in terms of weight savings and relative performance among laminates and materials. Later on, a Matlab version of Lam search became available. More capabilities such as buckling were added.

The most important advance, however, is the universal conversion of any FEA output of a component subjected to multiple loads to find the best DD as one material in one zone for the component. It can be applied to any existing component to find the best DD and the QI laminates, their relative strength ratios (safety factors), and weight. It will also find the taper (thickness reduction of lightly loaded locations), therefore the weight savings. Lam search takes Tsai's modulus and DD from interesting research discoveries to readily utilizable methods in the industry. All the potential manufacturing advantages that are offered by DD can be examined to determine the potential direct and indirect cost savings.

7. Conclusions

DD is a science-based discovery that is more than a research idea: it is ready to be used in the industry to save weight. It started with Tsai's modulus as the one and only material constant that is needed for design of composite structures. It is therefore as straightforward as metals.

DD can save weight and cost because it is a more effective laminate. Can achieve desired properties with 2- or 4-ply sub-laminates that would not be possible with 12-, 16- and 20-ply for quad.

With thinner sub-laminate, the minimum gage is reduced proportionally, that opens up applications too thick for the traditional quad. More importantly, thinner sub-laminates can become homogenized faster. Not only manufacturing cost will reduce, DD can do one ply drop at a time, not two with mid-plane symmetry. Then ply drop can be positioned at any place, including on the exterior surfaces or on the neutral axis.

The most important opportunity, however, is to reduce weight with aggressive tapering. DD can do it but quad cannot.

DD can be made into one multi-axial non-crimp fabric. Then one axis layup will be sufficient. The speed increase in layup will be 4 to 6 times faster than the 4-axis layup of quad.

Lam search is a tool that can do direct conversion of any FEA model of a component from whatever material used to the best DD, and its final weight based on aggressive taper. The final structure will fully integrate the design features and the manufacturing process that are reflected in the same FEA model.

Without DD the traditional quad will continue to present challenges to manufacturing to produce a component that may deny many of the original design features, and with costly and time-consuming processes.

A proposed program will be in collaboration with Stanford University and their pending patent on double-double, and subsequent proprietary information on the card sliding method and other manufacturing advances.

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The data presented in Figure 1 were based on information from JIA Liyong, of the Aviation Industry Corporation of China.

Data availability

The raw data required to reproduce these findings are available on request. The processed data required to reproduce these findings are available on request.

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